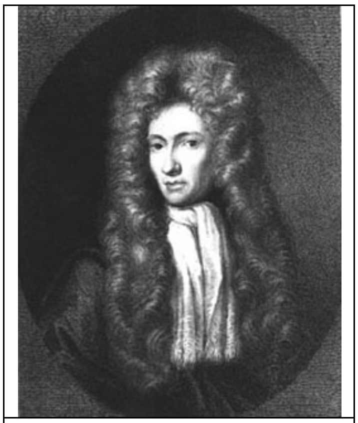
Annex A
The Ideal Gas Law



Robert Boyle (1627-1691)

Joseph Louis Gay-Lussac (1778-1850)

Amedeo Avogadro (1776-1856)

In 1660 the Irish chemist Robert Boyle, known as the "Father of Modern Chemistry", presented one of the first quantitative treatments of the gas. In 1802, the French physicist and chemist Joseph Louis Gay-Lussac established another equation for gases and vapors. The combination of these two laws resulted in the well known Ideal Gas Law.

Consider a portion of the physical space with a volume V, containing N molecules of a substance (gas or vapor) at a temperature T°K and pressure p. Then:

$$\boxed{pV = NkT} \qquad (1)$$

where:

p = pressure of the gas (atm, mmHg, kPascal etc.)
V = volume occupied by the gas (cubic meters, liters etc.)
N = number of gas or vapor molecules within the volume V
T = absolute temperature of the gas in °K
k = Boltzmann's constant = 1.38×10^{-23} joules / °K

Considering that the use of the number of molecules N is not practical, the formula was modified, based on the findings of the Italian professor of chemistry, Amedeo Avogadro.

In 1811, he discovered two interesting properties (that took almost 50 years to be accepted):

1. A quantity M of any substance, M being the molar mass expressed in grams, always contains $N_0 = 6.023 \times 10^{23}$ molecules.

From this property, and considering that the mass m is proportional to the number of molecules N, it may be written:

$$\boxed{N = \frac{m}{M} N_0} \qquad (2)$$

where the number M is called "one mol", or "one gram-molecule" of the substance, and the fraction (m / M) is called "the number of moles" of the substance. For example, one mol of water is equal to 18 grams, and 36 grams of water equals 2 moles of water.

2. N_0 molecules of any gas or vapor always occupies a volume V_0 = 22.4150 liters, at a pressure p_0 = 1 atmosphere and a temperature T_0 = 273.15 °K.

From this property:

$$\boxed{p_0\,V_0 = N_0\,k\,T_0 \quad \text{or} \quad k = \frac{p_0\,V_0}{N_0\,T_0}} \qquad (3)$$

By substituting equations (2) and (3) into (1), another expression of the Ideal Gas Law is obtained:

$$p\,V = \left(\frac{m}{M}N_0\right)\left(\frac{p_0\,V_0}{N_0\,T_0}\right)T$$

$$\boxed{p\,V = \frac{m}{M}\,R\,T} \qquad (4)$$

where

R = Universal Gas Constant =
= p_0 V_0 / T_0 =
= (1 atm) (22.415 liters) / 273.15 ºK =
= 0.08206 (atm-liters / ºK) =
= 8.314 joules / ºK

For example, the mass of oxygen (O_2) within a reservoir with a volume V = 56.6 liters, at a pressure p = 1.37 atm and a temperature of 27ºC (or T = 300.15ºK), is 100.74 g:

p = 1.37 atm
V = 56.6 liters
T = 273.15 + 27 = 300.15 ºC
M = 32 g of O_2 (molecular mass always taken in grams)
R = 0.08206 liters-atm / ºC

$$m = \frac{p\,V\,M}{R\,T} = \frac{(1.37\text{atm})(56.6\,\text{liters})\,32\,(\text{g})}{0.08206\,(\text{liters atm}/{}^{o}\text{C})\,300.15\,({}^{o}\text{C})} = 100.74\,\text{g}$$

Note: To ensure correctness, a check of the units should be carried out. In this example, all units, except "grams", cancel out.

Another example: The density of a gas in standard conditions (1 atm and 0ºC) is 2.85796 grams/liter. To determine its molecular mass (or "molecular weight"), an usual method used in chemistry:

$$M = \frac{m}{V}\frac{R\,T}{p} = 2.85796\,(\text{g}/\text{liter})\,\frac{0.08206\,(\text{liters atm}/{}^{o}\text{C})\,273.15\,({}^{o}\text{C})}{(1\text{atm})} = 64.06\,\text{g}$$

This value corresponds to the gas is SO_2.

In general, at pressures and temperatures near the "standard conditions" (1 atm and 0ºC), all gases follow the "Ideal Gas Law", and they are said to behave like "perfect gases" or "ideal gases". Water vapor is one example of a perfect gas near the standard conditions.

NOTE: Actually, the name "perfect gas" is not adequate. It is not the gas that becomes "imperfect" under conditions away from the stardard ones. It is the model that is imperfect to describe the behavior of the gas under a wide range of conditions.