4. How a psychrometer works

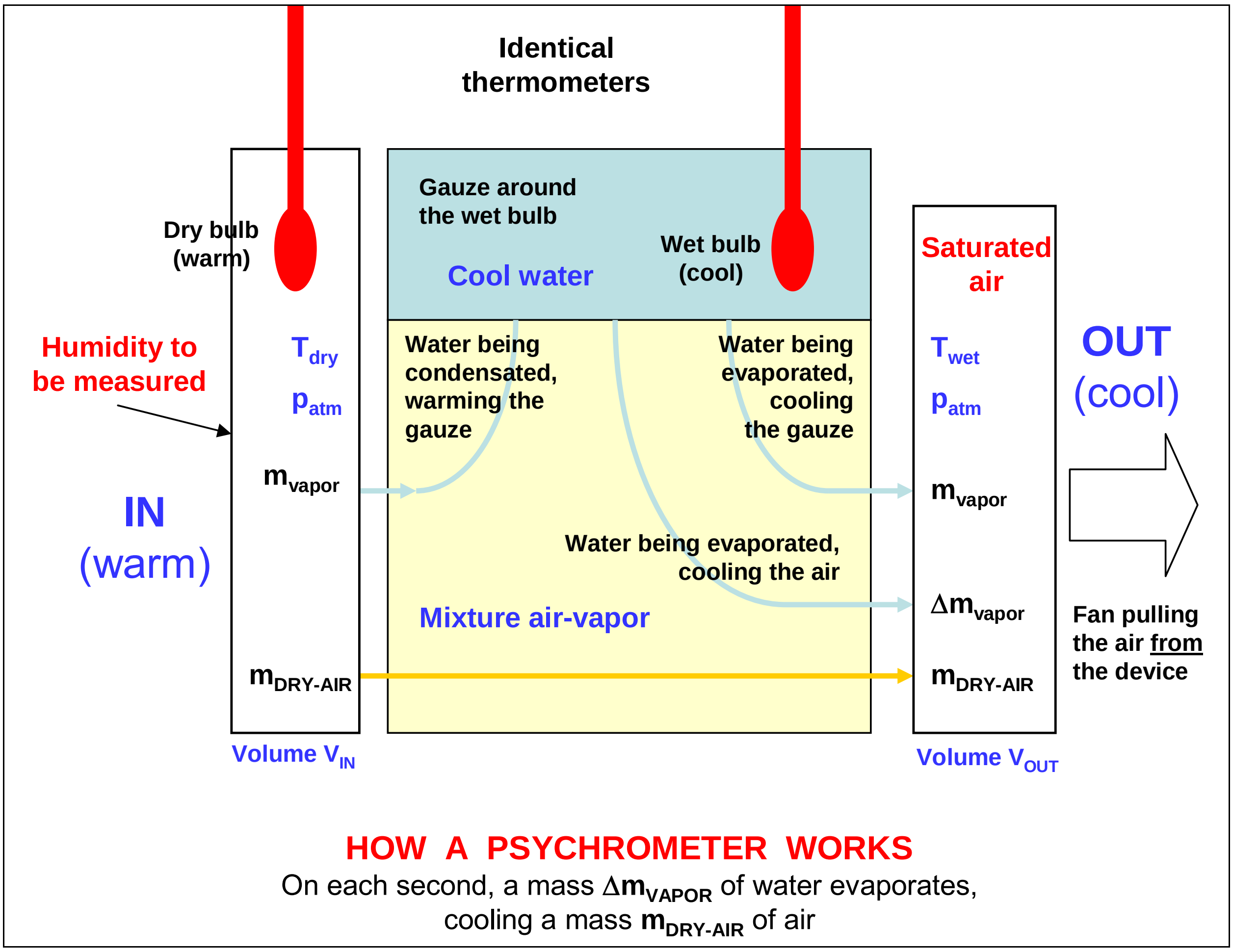

Figure 1: How a psychrometer works

Figure 5 shows in details how a psychrometer works. Each second, a net mass of water $\Delta \mathbf{m}_{\mathbf{vapor}}$ is evaporated, cooling a mass of air $\mathbf{m}_{\mathbf{air}}$.

In mathematical terms, according to the first principle of thermodynamics (conservation of energy):

Heat to vaporize the water = Heat from the cooling air

$$C_{VAPOR} \; \Delta m_{VAPOR} = m_{DRY-AIR} \; C_{DRY-AIR-P} \; \Delta T \qquad (1)$$

where:

$\Delta T = T_{DRY} - T_{WET}$ ("Wet Bulb Depression")
C_{VAPOR} = mean latent heat of vaporization of water from o to 50ºC = 584 cal / gram of water
Δm_{VAPOR} = mass of liquid water being vaporized = to be determined
$m_{DRY-AIR}$ = mass of air being cooled = to be determined
$C_{DRY-AIR-P}$ = sensible heat capacity of air at ct. pressure = 0.24019 cal / gram / ºC

T = absolute temperature of mixture air-vapor (dry bulb temperature) = 273.15 + t ºC
T_{WET} = absolute temperature of wet bulb = 273.15 + t_{WET} ºC

All that follows is a consequence of this simple equation (1).

C_{VAPOR} changes with temperature (see Fig. 6), but since in the range 0-50ºC the change is relatively small, to simplify the calculations the average value is used. In the same range, $C_{DRY\text{-}AIR\text{-}P}$ is practically constant.

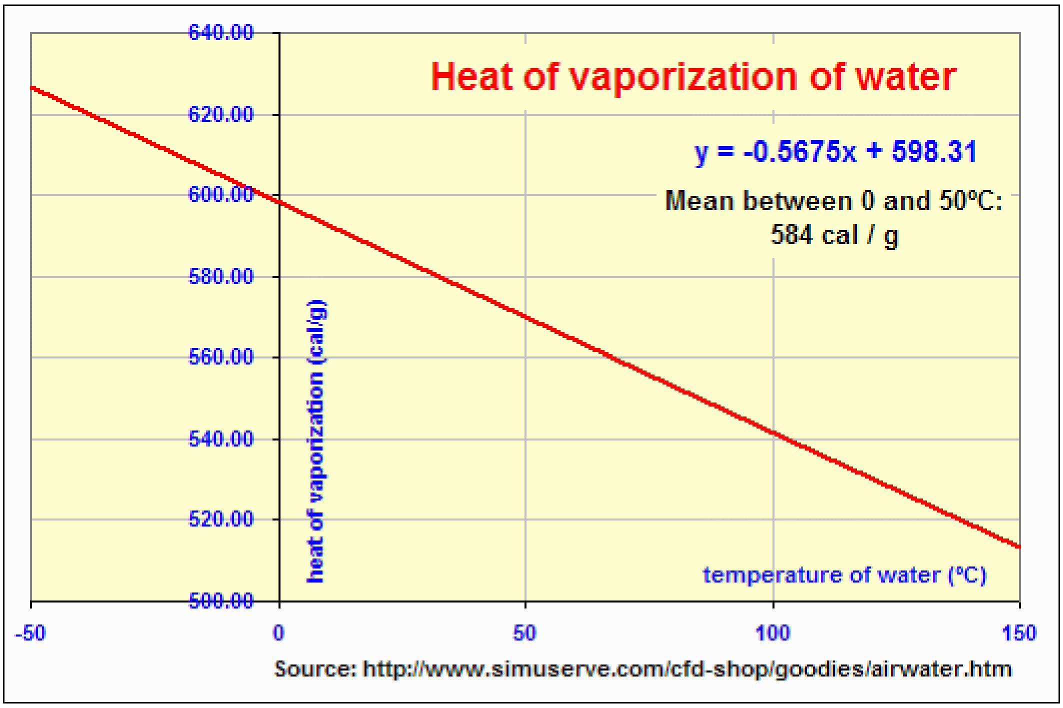

Figure 2: Latent Heat of Vaporization of Water

Equation (1) may also be written as:

$$\frac{\Delta m_{VAPOR}}{m_{AIR}} = \frac{C_{AIR-P}\ \Delta T}{C_{VAPOR}} \qquad (2)$$

In the volume V_{IN} (Fig. 5), there is a mixture of $\mathbf{m}_{\mathbf{DRY\text{-}AIR}}$ grams of dry air and $\mathbf{m}_{\mathbf{VAPOR}}$ grams of water vapor. According to the Ideal Gas Law (Annex A):

$$\frac{m_{VAPOR}}{m_{DRY-AIR}} = \left(\frac{p_{VAPOR}\ V_{IN}\ M_{VAPOR}}{R\ T_{DRY}}\right)\left(\frac{R\ T_{DRY}}{p_{DRY-AIR}\ V_{IN}\ M_{DRY-AIR}}\right) = \left(\frac{p_{VAPOR}}{p_{DRY-AIR}}\right)\left(\frac{M_{VAPOR}}{M_{DRY-AIR}}\right) \qquad (3)$$

Using the Dalton Law (Annex B) in volume V_{IN}

$$p_{ATM} = p_{DRY\text{-}AIR} + p_{VAPOR}$$

and defining the Humidity Ratio HR (or Absolute Humidity):

$$HR = m_{VAPOR} / m_{DRY\text{-}AIR} \qquad (4)$$

equation (3) becomes

$$HR = \frac{m_{VAPOR}}{m_{DRY-AIR}} = \left(\frac{M_{VAPOR}}{M_{DRY-AIR}}\right)\left(\frac{p_{VAPOR}}{p_{ATM} - p_{VAPOR}}\right) \tag{5}$$

where

M_{VAPOR} = molecular mass of water vapor = 18 grams/mol
$M_{DRY\text{-}AIR}$ = molecular mass of dry air (weighted average) = 29 grams/mol
p_{ATM} = local atmospheric pressure (mmHg)
p_{VAPOR} = partial pressure of vapor at T_{DRY} (to be determined)

The molecular mass of dry air is calculated as follows:

Molecule	% Vol	Mol	Product
Nitrogen (N_2)	78.09	28.013	21.88
Oxygen (O_2)	20.95	31.998	6.70
Argon (A)	0.93	39.948	0.37
Carbon Dioxide (CO_2)	0.03	43.999	0.01
Sum	**100.00**		**28.96**
1 Mol of Dry Air			**28.96**

Using the same reasoning in volume V_{OUT} (Fig. 5), a similar relationship is found:

$$\frac{m_{VAPOR} + \Delta m_{VAPOR}}{m_{DRY-AIR}} = HR + \frac{\Delta m_{VAPOR}}{m_{DRY-AIR}} = \left(\frac{M_{VAPOR}}{M_{DRY-AIR}}\right)\left(\frac{p_{SAT}(T_{WET})}{p_{ATM} - p_{SAT}(T_{WET})}\right) \tag{6}$$

where $\mathbf{p_{SAT}}$ is the saturation pressure of water vapor at T_{WET} (Annex C).

Subtracting Eq. (6) from Eq. (5) and equating it to Eq. (2) results:

$$\frac{p_{SAT}(T_{WET})}{p_{ATM} - p_{SAT}(T_{WET})} - \frac{p_{VAPOR}}{p_{ATM} - p_{VAPOR}} = \left(\frac{C_{DRY-AIR-P}}{C_{VAP}}\frac{M_{DRY-AIR}}{M_{VAPOR}}\right)\Delta T \tag{7}$$

Defining the "Humidity Constant" **k** as:

$$k = \frac{C_{DRY-AIR-P}\ M_{DRY-AIR}}{C_{VAPOR}\ M_{VAPOR}} = \frac{0.24\,(cal/gram/^{o}C)\ 29\,(grams/mol)}{584\,(cal/gram)\ 18\,(grams/mol)} = \frac{0.000662}{^{o}C}$$

Note: cal / gram / ºC = cal / (gram ºC)

equation (7) becomes:

$$\frac{p_{SAT}(T_{WET})}{p_{ATM} - p_{SAT}(T_{WET})} - \frac{p_{VAPOR}}{p_{ATM} - p_{VAPOR}} = k\,\Delta T \tag{8}$$

In Eq. (8) the only unknown is p_{VAPOR} which is equal to:

$$p_{VAPOR} = p_{ATM} \frac{(1-k\,\Delta T)\,p_{SAT}(T_{WET}) - k\,\Delta T\,p_{ATM}}{(1-k\,\Delta T)\,p_{ATM} - k\,\Delta T\,p_{SAT}(T_{WET})} \quad (8)$$

where (Annex C):

$$p_S(t^o C) = (0.61078 * 7.501)\, e^{\left(\frac{17.2694 * t}{238.3 + t}\right)} \quad mmHg \quad (9)$$

Eq. (8) may be simplified, considering the numerical values of some of the terms:

1.

$\Delta T\ ^oK = \Delta t\ ^oC$ by the definition of the Kelvin scale.

2.

If

$0 < \Delta T < 20^oC$ and

$k = 0.000662\ /\ ^oC$

Then

$k\ \Delta T < 0.013 << 1$

So, $1 - k\ \Delta T\ \sim= 1.$

3.

If

$t_{WET} < 50^oC$ and

$p_{SAT}\ (50^oC) = 100$ mmHg and

$p_{ATM}\ (13000\ m) = 127\ mmHg < p_{ATM} < 760\ mmHg$

Then

$k\ \Delta T\ p_{SAT}\ (t_{WET}) < 1.3\ mmHg << 127\ mmHg < p_{ATM}$

So, $k\ \Delta T\ p_{SAT}\ (t_{WET}) \sim= 0.$

Simplifying eq. (8), the Psychrometric Equation becomes:

$$p_{VAPOR} = p_{SAT}(t_{WET}) - k\ \Delta t\ p_{ATM} \quad (10)$$

a well known relationship.

Finally, the Relative Humidity (RH) (see Annex C) may be calculated as:

$$RH\ \% = 100\ \frac{p_{VAPOR}}{p_{SAT}(t_{DRY})} \quad (11)$$

OBS: It may be noted that the so called "Psychrometric Constant"

$$\gamma = \frac{p_{ATM}\, C_{AIR-P}}{\varepsilon\, C_{VAP}}$$

was not used here because it is not a constant (depends on p_{ATM}); ($k\ p_{ATM}$) was used instead.

Example:

When the local atmospheric pressure is 760 mmHg (sea level), the temperature of the dry bulb is 20ºC and the wet bulb is at 15ºC, the relative humidity is 58.6%:

$$p_S(20^o C) = (0.61078 * 7.5)\ e^{\left(\frac{17.2694 * 20}{238.3+20}\right)} = 17.44\,\text{mmHg}$$

p_S (15ºC) = 12.74 mmHg
$k\ \Delta T\ p_{ATM}$ = 0.000662 (1/ºC) * 5ºC * 760 mmHg = 2.52 mmHg
p_{VAPOR} = 12.74 - 2.52 = 10.22 mmHg
RH% = 100 * (10.22 mmHg) / (17.44 mmHg) = 58.6 %

Note: 1 kPascal = 7.501 mmHg

Precision of the measurement when using expression (10):

To reduce the measurement error, three precautions must be taken:

1. The psychrometer must be insulated from the radiant heat of the environment (infrared rays). This may be accomplished by surrounding the wet bulb with a reflective surface (e.g. aluminum foil), but leaving a space for ventilation between the foil and the gauze.

2. Although ideally distilled water should be used to moisten the gauze, in practice, using filtered drinking water may be sufficient.

3. The wet bulb must be ventilated, to avoid the formation of a saturated-vapor layer around the wet bulb, which reduces the evaporation, thus increasing its temperature. Ventilation may be set around one meter per second. As a reference, a small fan (38mm x 38mm x 10mm) for a 486-CPU, operates at an air-speed of 1.6 meters per second (this fan costs around US$ 12).

Here is a real example using lab data on given **date X**:

Site altitude: 637 meters (from http://www.calle.com/world/index.html)
Dew point temperature (measured): 8ºC
Dry bulb temperature: 18.5ºC (ambient temperature)
Wet bulb temperature: 15.0ºC (Augustus Psychrometer, no ventilation)
Wet bulb temperature: 12.5ºC (Augustus Psychrometer, ventilated 1 meter/second)
Commercial electronic humidity meter (capacitive): 51%

Calculated relative humidity (using "Quick_Start.xls"):

Using psychrometer, no ventilation: 69.9% (error)
Using psychrometer, with ventilation: 50.5% (no error)
Using Dew Point and ambient temperatures: 50.5% (reference)

Here is another real example using lab data on given **date Y**:

Site altitude: 637 meters (from http://www.calle.com/world/index.html)
Dew Point temperature (measured): 9.9ºC
Dry bulb temperature: 25.3ºC (ambient temperature)
Wet bulb temperature: 17.5ºC (Augustus Psychrometer, no ventilation)
Wet bulb temperature: 16.0ºC (Augustus Psychrometer, ventilated 1 meter/second)
Commercial electronic humidity meter (capacitive): 36%

Calculated relative humidity (using "Quick_Start.xls"):

Using psychrometer, no ventilation: 47.0% (error)
Using psychrometer, with ventilation: 38.4% (small error)
Using Dew Point and ambient temperatures: 38.0% (reference)

NOTE: All the equations and the calculations used in this article were developed and tested with Mathematica 5.0 (Wolfram Research, www.wolfram.com), a highly recommended software.